

Element spots in Ap and Hg-Mn stars from current-driven diffusion

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Abstract. The stars of the middle main sequence often have spot-like chemical structures at their surfaces. We consider the diffusion process caused by electric currents that can lead to the formation of such chemical spots. Diffusion was considered using partial momentum equations derived by the Chapman-Enskog method. We argue that diffusion caused by electric currents can substantially change the surface chemistry of stars and form spotted chemical structures even in a relatively weak magnetic field. The considered mechanism can be responsible for a formation of element spots in Hg-Mn and Ap-stars.

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1. Introduction

Diffusion can lead to evolution of atmospheric chemistry in stars and be the reason of chemical peculiarities. This particularly concerns the stars of the middle main sequence that often have relatively quiescent surface layers. Many stars with peculiar chemical abundances show line variations caused by element spots on their surface (see, e.g., Pyper 1969, Khokhlova 1985, Silvester et al. 2012). It was thought that chemical spots can only occur in the presence of a strong magnetic field. Indeed, some Ap stars show variations of both spectral lines and magnetic field strength that can be related to rotation of chemical and magnetic spots. Often such stars have the strongest concentration of heavy elements around the magnetic poles (see, e.g., Havnes 1975). Note that a reconstruction of the stellar magnetic geometry from observations is a very complex problem. The magnetic Doppler imaging code developed by Piskunov & Kochukhov (2002) makes it possible to derive the magnetic map of a star self-consistently with the distribution of the chemical elements. The reconstructions show that the magnetic and chemical maps of stars can be very complex (Kochukhov et al. 2004a) and usually chemical elements do not exhibit a correlation with the magnetic geometry. The calculated distributions demonstrate the complexity of diffusion in Ap-stars and show that chemical distributions are affected by a number of poorly understood phenomena and are not directly related to the strength of the magnetic field.

Often, the chemical spots on the surface of stars are related to anisotropic diffusion in the magnetic field. Indeed, the magnetic field of Ap-stars ($B \sim 10^3 - 10^4$ G) is suffi-

ciently strong to magnetize plasma and make diffusion anisotropic. Anisotropy of diffusion is characterized by the Hall parameter, $x = \omega_{Be}\tau_e$, where $\omega_{Be} = eB/m_e c$ is the gyrofrequency of electrons and τ_e is their relaxation time. If the background plasma is hydrogen, then $\tau_e = 3\sqrt{m_e}(k_b T)^{3/2}/4\sqrt{2}\pi e^4 n \Lambda$ (see, e.g., Spitzer 1998) where n and T are the number density of electrons and their temperature, Λ is the Coulomb logarithm. If $x \geq 1$, the rates of diffusion along and across the magnetic field are different and diffusion can lead to inhomogeneous element distributions. The condition $x \geq 1$ yields the following estimate of the magnetic field that magnetizes plasma

$$B \geq B_e = 2.1 \times 10^3 n_{15} T_4^{-3/2} \Lambda_{10} \text{ G}, \quad (1)$$

where $\Lambda_{10} = \Lambda/10$, $n_{15} = n/10^{15}$, and $T_4 = T/10^4$ K. Some stars with chemical spots have such a strong magnetic field and diffusion can be anisotropic there.

In recent years, however, the discovery of chemical inhomogeneities in the so-called Hg-Mn stars has raised some doubts regarding their magnetic origin. The aspect of spot-like chemical structures in HgMn stars was discussed first by Hubrig & Mathys (1995). In contrast to Ap-stars, no strong magnetic field of kG order has ever been detected in HgMn stars. For instance, Wade et al. (2004) find no longitudinal field above 50 G in the brightest Hg-Mn star α And with chemical spots at the surface. The authors establish an upper limit of the global field at ≈ 300 G that is not sufficient to magnetize plasma. Weak magnetic fields in the atmospheres of Hg-Mn stars have been detected by a number of authors (see, e.g., Hubrig & Castelli 2001, Hubrig et al. 2006, Makaganiuk et al. 2011, 2012). In a recent study by Hubrig et al. (2012),

the previous measurements of the magnetic field have been re-analysed and the presence of a weak longitudinal magnetic field up to 60-80 G have been revealed in several HgMn stars. The complex interrelations between the magnetic field and the chemical structures show how incomplete is our understanding of diffusion processes in stars.

In this paper, we consider one more diffusion mechanism that contributes to a formation of chemical spots in stars. This mechanism is relevant to electric currents and has not been studied in stellar conditions yet. We concentrate on the main qualitative features of this process and compare the diffusion rate caused by the presence of electric currents and the rate of other diffusion processes. We show that interaction of the electric current with ions leads to diffusion in the direction perpendicular to the both electric current and magnetic field. Such diffusion can alter the surface chemical distributions at a substantially weaker magnetic field than B_e .

2. Basic equations

Consider a cylindrical plasma configuration with the magnetic field parallel to the axis z , $\mathbf{B} = B\mathbf{e}_z$; (s, φ, z) and $(\mathbf{e}_s, \mathbf{e}_\varphi, \mathbf{e}_z)$ are cylindrical coordinates and the corresponding unit vectors. We assume that the magnetic field depends on the cylindrical radius alone, $B = B(s)$. Then, the electric current is

$$j_\varphi = -(c/4\pi)\partial B/\partial s. \quad (2)$$

We suppose that $j_\varphi \rightarrow 0$ at large s and, hence, $B \rightarrow B_0 = \text{const}$ at $s \rightarrow \infty$. Note that $B(s)$ can not be an arbitrary function of s because, generally, the magnetic configurations can be unstable for some dependences of $B(s)$ on s (see, e.g., Tayler 1973, Bonanno & Urpin 2008a,b for more detail). The timescale of this instability is usually shorter than the diffusion timescale and, therefore, a formation of chemical structures in such magnetic configurations is unlikely.

We assume that plasma consists of electrons e , protons p , and a small admixture of heavy ions i . The number density of species i is such small that it does not influence dynamics of plasma. Therefore, this species can be treated as test particles that interacts only with electrons and background hydrogen plasma. The hydrostatic equilibrium reads

$$-\nabla p + \rho \mathbf{g} + \frac{1}{c} \mathbf{j} \times \mathbf{B} = 0, \quad (3)$$

where p and ρ are the pressure and density, respectively, $\mathbf{g} = -g\mathbf{e}_z$ is gravity. Since the background plasma is hydrogen, $p \approx 2nk_B T$ where k_B is the Boltzmann constant. Integrating the s -component of Eq. (3), we obtain

$$n = n_0 \left(\frac{T_0}{T} \right) \left(1 + \frac{1}{\beta_0} - \frac{1}{\beta} \right), \quad (4)$$

where $\beta = 8\pi p_0/B^2$; (p_0, n_0, T_0, β_0) are the values of (p, n, T, β) at $s \rightarrow \infty$.

The partial momentum equations in fully ionized multi-component plasma has been considered by a number of authors (see, e.g., Urpin (1981)). If the mean hydrodynamic

velocity is zero and only small diffusive velocities are non-vanishing, the partial momentum equation for the species i can be written as

$$-\nabla p_i + Z_i e n_i \left(\mathbf{E} + \frac{\mathbf{V}_i}{c} \times \mathbf{B} \right) + \mathbf{R}_{ie} + \mathbf{R}_{iH} + \mathbf{F}_i = 0, \quad (5)$$

where Z_i is the charge number of the species i , p_i , and n_i are its partial pressure and number density, \mathbf{V}_i is its velocity, and \mathbf{E} is the electric field. The force \mathbf{F}_i is the external force on species i ; in stars, \mathbf{F}_i is usually determined by gravity \mathbf{g} and the radiation force. The forces \mathbf{R}_{ie} and \mathbf{R}_{iH} are caused by the interaction of ions i with electrons and protons, respectively. Note that forces \mathbf{R}_{ie} and \mathbf{R}_{iH} are internal and their sum over all plasma components is zero in accordance with Newton's third law. Since diffusive velocities are typically small, we neglect the terms proportional $(\mathbf{V}_i \cdot \nabla) \mathbf{V}_1$ in the momentum equation (5).

The s - and φ -components of Eq.(5) yield

$$-\frac{d}{ds}(n_i k_B T) + Z_i e n_i \left(E_s + \frac{V_{i\varphi}}{c} B \right) + R_{ies} + R_{iHs} = 0, \quad (6)$$

$$Z_i e n_i \left(E_\varphi - \frac{V_{is}}{c} B \right) + R_{ie\varphi} + R_{iH\varphi} = 0 \quad (7)$$

The force \mathbf{R}_{ie} is caused by scattering of ions i on electrons. If n_i is small compared to the number density of protons, \mathbf{R}_{ie} is given by

$$\mathbf{R}_{ie} = -\frac{Z_i^2 n_i}{n} \mathbf{R}_e \quad (8)$$

where \mathbf{R}_e is the force acting on the electron gas (see, e.g., Urpin 1981). Since $n_i \ll n$, \mathbf{R}_e is determined mainly by scattering of electrons on protons but scattering on ions i gives a small contribution. Therefore, we can use for \mathbf{R}_e the expression for one component hydrogen plasma calculated by Braginskii (1965). In our model of a cylindrical plasma configuration, this expression reads

$$\mathbf{R}_e = -\alpha_\perp \mathbf{u} + \alpha_\parallel \mathbf{b} \times \mathbf{u} - \beta_\perp^{uT} \nabla T - \beta_\parallel^{uT} \mathbf{b} \times \nabla T, \quad (9)$$

where $\mathbf{u} = -\mathbf{j}/en$ is the difference between the mean velocities of electrons and protons, $\mathbf{b} = \mathbf{B}/B$, α_\perp , α_\parallel , β_\perp^{uT} , and β_\parallel^{uT} are coefficients calculated by Braginskii (1965). The first two terms on the r.h.s. of Eq.(9) describe the standard friction force caused by a relative motion of the electron and proton gases. The last two terms on the r.h.s. of Eq.(9) represent the so-called thermoforce caused by a temperature gradient. This part of \mathbf{R}_e is responsible for thermodiffusion.

Taking into account Eq.(2), we have

$$\mathbf{u} = \frac{c}{4\pi en} \frac{dB}{ds} \mathbf{e}_\varphi. \quad (10)$$

In this paper, we consider diffusion only in a relatively weak magnetic field that does not magnetize electrons, $x \ll 1$. Substituting Eq.(10) into Eq.(8) and using coefficients α_\perp , α_\parallel , β_\perp^{uT} , and β_\parallel^{uT} with the accuracy in linear terms in x , we obtain

$$R_{ie\varphi} = Z_i^2 n_i \left(0.51 \frac{m_e}{\tau_e} u + 0.81 x k_B \frac{dT}{ds} \right), \quad (11)$$

$$R_{ies} = Z_i^2 n_i \left(0.21 x \frac{m_e}{\tau_e} u + 0.71 k_B \frac{dT}{ds} \right). \quad (12)$$

The force \mathbf{R}_{iH} is the sum of two terms as well, $\mathbf{R}_{iH} = \mathbf{R}'_{iH} + \mathbf{R}''_{iH}$ that are proportional to the relative velocity of ions i and protons and the temperature gradient, respectively. The friction force \mathbf{R}'_{iH} can be easily calculated if $A_i = m_i/m_p \gg$

1. In this case, $\mathbf{R}'_{iH} \propto (\mathbf{V}_H - \mathbf{V}_i)$ but taking into account that the mean velocity of the background plasma is zero in our model, the friction force can be represented as (see, e.g., Urpin 1981)

$$\mathbf{R}'_{iH} \approx \frac{0.42m_i n_i Z_i^2}{\tau_i} (-\mathbf{V}_i), \quad (13)$$

where $\tau_i = 3\sqrt{m_i}(k_B T)^{3/2}/4\sqrt{2\pi}e^4 n \Lambda$ and τ_i/Z_i^2 is the timescale of ion-proton scattering; we assume that Coulomb logarithms are the same for all types of scattering.

The thermal part of the friction force, \mathbf{R}''_{iH} , has been calculated by Urpin (1981). Since there is no diffusion in the z -direction, the expression for \mathbf{R}''_{iH} with accuracy in linear terms in magnetization can be written as

$$\mathbf{R}''_{iH} = Z_i^2 n_i k_B (1.71 \nabla T - 3.43 y \mathbf{b} \times \nabla T), \quad (14)$$

where

$$y = \frac{eB\tau_p}{m_p c}, \quad \tau_p = \frac{3\sqrt{m_p}(k_B T)^{3/2}}{4\sqrt{2\pi}e^4 n \Lambda}. \quad (15)$$

Then, the cylindrical components of \mathbf{R}''_{iH} are

$$R_{iHs} = Z_i^2 n_i \left(-\frac{m_i}{\tau_i} V_{is} + 1.71 k_B \frac{dT}{ds} \right), \quad (16)$$

$$R_{iH\varphi} = Z_i^2 n_i \left(-\frac{m_i}{\tau_i} V_{i\varphi} - 3.43 y k_B \frac{dT}{ds} \right). \quad (17)$$

The momentum equation for the species i (Eq.(5)) depends on cylindrical components of the electric field, E_s and E_φ . These components can be determined from the momentum equations of electrons and protons

$$-\nabla(nk_B T) - en \left(\mathbf{E} + \frac{\mathbf{u}}{c} \times \mathbf{B} \right) + \mathbf{R}_e = 0, \quad (18)$$

$$-\nabla(nk_B T) + en\mathbf{E} - \mathbf{R}_e + \mathbf{F}_p = 0. \quad (19)$$

Taking into account the condition (3) and the friction force \mathbf{R}_e (Eq. (9)) calculated by Braginskii (1965), we obtain with accuracy in linear terms in x

$$E_s = -\frac{uB}{2c} - \frac{1}{e} \left(0.21 \frac{m_e u}{\tau_e} x + 0.71 k_B \frac{dT}{ds} \right), \quad (20)$$

$$E_\varphi = -\frac{1}{e} \left(0.51 \frac{m_e u}{\tau_e} + 0.81 x k_B \frac{dT}{ds} \right). \quad (21)$$

Substituting Eqs.(11)-(12), (16)-(17), and (20)-(21) into Eqs.(6) and (7), we arrive at the expression for a diffusion velocity, \mathbf{V}_i . The radial component of this velocity reads

$$V_{is} = V_{ni} + V_T + V_B, \quad (22)$$

where

$$V_{ni} = -D \frac{d \ln n_i}{ds}, \quad V_T = D_T \frac{d \ln T}{ds}, \quad V_B = D_B \frac{d \ln B}{ds}; \quad (23)$$

V_{ni} and V_T are the velocities of ordinary diffusion and thermodiffusion, respectively, V_B is the diffusive velocity associated with the electric current. The diffusion coefficients are

$$D = \frac{2.4c_i^2 \tau_i}{Z_i^2}, \quad D_T = \frac{2.4c_i^2 \tau_i}{Z_i^2} (2.42 Z_i^2 - 0.71 Z_i - 1), \quad (24)$$

$$D_B = \frac{2.4c_A^2 \tau_i}{A_i Z_i} (0.21 Z_i - 0.71),$$

where $c_i^2 = k_B T/m_i$ and $c_A^2 = B^2/4\pi m_p n$. Eq.(22) describes the drift of ions i under the combined influence of ∇n_i , ∇T , and \mathbf{j} .

The diffusive velocity given by Eq. (22) differs from the standard expression used in astrophysical calculations (see, e.g., Chapman & Cowling 1970, Burgers 1969) by the presence of a term $\propto dB/ds$. It follows from our consideration that this term is caused by scattering of heavy ions on electrons. The classical works by Chapman & Cowling (1970) and Burgers (1969) derive the atomic diffusion coefficients from the Boltzmann equation but these coefficients are better suited to diffusion in neutral gases or plasma with a large charge of the background ions. The point is that these studies neglect scattering of impurities on electrons in plasma, and take into account their scattering only on the background ions. The latter is correct if the charge of background ions, Z_0 , is large, $Z_0 \gg 1$. In stellar plasmas, however, the main background ions are usually protons and, hence, $Z_0 = 1$. Therefore, neglecting the contribution of electrons into kinetic processes is unjustified. This fact was first clearly understood by Braginskii (1965) in his theory of transport phenomena in a high-temperature plasma. This result can be clarified by simple qualitative estimates. Indeed, the momentum of electrons is $\sim m_e c_e$ ($c_e = \sqrt{k_B T/m_e}$ is the thermal velocity of electrons), and the rate of momentum transfer by electrons to impurities is $\sim m_e c_e \nu_e \sim m_e c_e / \tau_e$ where ν_e is the frequency of electron collisions. On the other hand, the momentum of protons is $\sim m_p c_p$ where $c_p = \sqrt{k_B T/m_p}$ is the thermal velocity of protons and, correspondingly, the rate of momentum transfer by protons is $\sim m_p c_p \nu_p \sim m_p c_p / \tau_p$ where ν_p is the frequency of proton collisions. Comparing these expressions, we obtain that the rates of momentum transfer by electrons and protons are of the same order and, hence, neglecting the electron contribution is unjustified in plasma with $Z_0 \sim 1$. However, if the background plasma consists of ions with the charge $Z_0 \gg 1$, then one should replace τ_p by the relaxation time of the background ions that is $\sim \tau_p/Z_0^4$. In this case, the rate of momentum transfer by ions turns out to be $\sim Z_0^4$ times greater than that by electrons. If $Z_0 \gg 1$, the electron contribution is small and can be neglected. Therefore, the classical diffusion theory is justified in this case.

The fact that the consistent consideration of scattering on electrons leads to diffusion of heavy ions with the velocity $\propto dB/ds$ is well known in plasma physics and was first discussed by Vekshtein et al. (1975). This process plays an important role in diffusion of impurities from the walls of the discharge chamber and diaphragms in a dense plasma in tokamaks (see, e.g., Vekshtein 1987 for review). Even a small fraction of impurity ions can considerably affect the radiation, electrical conductivity, and other plasma parameter. Unfortunately, this effect is usually neglected in studies of diffusion in stars but we will show that it can play an important role in a spot formation, particularly, in weakly magnetized stars.

3. Distribution of ions in the presence of electric currents

Consider the equilibrium distribution of elements in our model. In equilibrium, we have $V_{is} = 0$ and Eq.(22) yields

$$\frac{d \ln n_i}{ds} = \frac{D_T}{D} \frac{d \ln T}{ds} + \frac{D_B}{D} \frac{d \ln B}{ds}. \quad (25)$$

The second term on the r.h.s. is caused by the presence of electric currents and describes the current-driven diffusion. Note that this type of diffusion is driven by the electric current rather than an inhomogeneity of the magnetic field. Occasionally, the conditions $dB/ds \neq 0$ and $j \neq 0$ are equivalent in our simplified model. Equation of hydrostatic equilibrium (3) yields

$$\frac{d}{ds}(nk_B T) = -\frac{B}{8\pi} \frac{dB}{ds}. \quad (26)$$

Substituting expression (26) into Eq.(25) and integrating, we obtain

$$\frac{n_i}{n_{i0}} = \left(\frac{T}{T_0}\right)^\nu \left(\frac{n}{n_0}\right)^\mu, \quad (27)$$

where

$$\begin{aligned} \nu &= 2Z_i^2 + 0.71Z_i - 1, \\ \mu &= -2Z_i(0.21Z_i - 0.71), \end{aligned} \quad (28)$$

where n_{i0} is the value of n_i at $s \rightarrow \infty$. Denoting the local abundance of the element i as $\gamma_i = n_i/n$ and taking into account Eq. (4), we have

$$\frac{\gamma_i}{\gamma_{i0}} = \left(\frac{T}{T_0}\right)^\nu \left(\frac{n}{n_0}\right)^{\mu-1} = \left(\frac{T}{T_0}\right)^{\nu-\mu+1} \left(1 + \frac{1}{\beta_0} - \frac{1}{\beta}\right)^{\mu-1}, \quad (29)$$

where $\gamma_{i0} = n_{i0}/n_0$. It turns out that the local abundance of ions is determined by both the temperature and magnetic field. The dependence of γ_i on T is very sensitive to the charge number of ions. For example, if $Z_i = 1$, the exponent in Eq.(29) is $\nu - \mu + 1 = 1.71$ but it is as large as 8.26 if $Z_i = 2$. Therefore, even a small change in the temperature can be the reason of a significant variation in the local abundance of chemical elements. If the magnetic field is constant then abundance anomalies are determined by the thermodiffusion alone. In this case, we have

$$\frac{\gamma_i}{\gamma_{i0}} = \left(\frac{T}{T_0}\right)^{2.42Z_i^2 - 0.71Z_i}. \quad (30)$$

Therefore, the regions with a higher temperature, $T > T_0$, should be overabundant by heavy elements but the regions with a lower temperature should be underabundant.

Local abundances are also flexible to the field strength and, particularly, this concerns very heavy ions. If variations of the temperature are negligible and $T \approx T_0$, then the distribution of elements is determined by the current-driven diffusion alone. In this case,

$$\frac{\gamma_i}{\gamma_{i0}} = \left(1 + \frac{1}{\beta_0} - \frac{1}{\beta}\right)^{\mu-1}. \quad (31)$$

Note that the exponent $(\mu - 1)$ is always negative if $Z_i \geq 3$ and, hence, heavy elements with $Z_i \geq 3$ are in deficit ($\gamma_i < \gamma_{i0}$) in the region with a weak magnetic field ($B > B_0$) but, on the contrary, such elements should be overabundant in the

spot where the magnetic field is weaker than the external field B_0 . The quantity $(\mu - 1)$ can reach large negative values and, therefore, dependence (31) on the magnetic field is very sharp. A combined influence of both thermo- and current-driven diffusion can result in a rather complicated distribution of elements.

4. Conclusion

We have considered diffusion of elements in the surface layers of stars under a combined influence of different diffusion mechanisms. A special attention was paid to the current-driven diffusion that has not been discussed yet in the context of chemical spots on stars. The diffusion velocity caused by electric current can be comparable or higher than the velocity of thermodiffusion. For instance, if electrons are not magnetized ($x < 1$) the velocities of thermo- and current-driven diffusions can be estimated as

$$V_T \sim \frac{5c_i^2 \tau_i}{L_T}, \quad V_B \sim \frac{c_A^2 \tau_i}{A_i L_B}, \quad (32)$$

where L_T and L_B are the lengthscales of T and B . The condition $V_B > V_T$ is satisfied if $c_A \gg 2c_i A_i^{1/2} (L_B/L_T)^{1/2}$ or

$$B > 2.6 \times 10^2 n_{15}^{1/2} T_4^{1/2} \left(\frac{L_B}{L_T}\right)^{1/2} \text{ G}, \quad (33)$$

where $n_{15} = n/10^{15} \text{ cm}^{-3}$ and $T_4 = T/10^4 \text{ K}$. It appears that even a relatively weak magnetic field ($\sim 10 - 100 \text{ G}$) can be the reason of current-driven diffusion with the velocity greater than that of thermodiffusion. From Eq. (32), one can estimate the velocity of current-driven diffusion as

$$V_B \sim 1.1 \times 10^{-4} A_i^{-1/2} B_4^2 n_{15}^{-2} T_4^{3/2} \Lambda_{10} L_{B10}^{-1} \text{ cm/s}, \quad (34)$$

where $\Lambda_{10} = \Lambda/10$, $B_4 = B/10^4 \text{ G}$, and $L_{B10} = L_B/10^{10} \text{ cm}$. The velocity V_B turns out to be sensitive to the field ($\propto B^2$) and, therefore, diffusion in a weak magnetic field requires a longer time to reach equilibrium.

The considered mechanism can form chemical spots even if the magnetic field is relatively weak whereas other diffusion processes produce spots only if the magnetic field is substantially stronger. For example, the radiative force and gravity can generally be responsible for chemical inhomogeneities in stars (see, e.g., Vauclair et al. 1979, Michaud et al. 1981). The radial diffusion velocity driven by these forces can be relatively large, and the distribution of impurities reaches a radial equilibrium on a short time scale (Michaud 1970). If the radiative and gravitational forces are of the same order of magnitude then the velocity of radial diffusion can be estimated as

$$V_r \sim g \tau_i \quad (35)$$

(see Vauclair et al. 1979). This velocity is typically greater than V_B in the surface layers of stars but the radial diffusion cannot form chemical spots if the radiative force and g have spherical symmetry. Departures from sphericity can be caused by the magnetic field since the diffusion velocity depends on its direction and strength. For instance, the radial diffusion velocities differ by a term of the order of

$$\Delta V \sim V_r (\omega_{Bi} \tau_i / Z_i^2)^2 \quad (36)$$

if the magnetic field is parallel and perpendicular to gravity; $\omega_{Bi} = eB/m_i c$ is the gyrofrequency of impurities (see, e.g., Vauclair et al. 1979, Alecian & Stift 2006). This difference in the radial velocities rather than V_r itself leads to formation of a spotted structure because spots cannot be formed if $\Delta V = 0$. Usually, ΔV is much smaller than V_r for more or less realistic stellar magnetic fields. For example, using calculations of Vauclair et al. (1979), one can estimate that ΔV is comparable to V_r if $B \sim 3 \cdot 10^4$ and $\sim 10^5$ G at the optical depth 1 and 10, respectively. These fields are even stronger than those detected in Ap-stars. In the case of Hg-Mn stars, the magnetic field is likely as weak as 10-100 G and, hence, ΔV is typically $\sim 10^8 - 10^6$ times smaller than V_r . Since ΔV turns out to be small, the velocity of current-driven diffusion can play an important role in real stars. The velocity V_B exceeds ΔV if the electric current satisfies the inequality

$$j / (cB/4\pi H) > A_i \frac{c_s^2}{c_A^2} (\omega_{Bi} \tau_i / Z_i^2)^2. \quad (37)$$

The parameter $\omega_i \tau_i$ is small in stars and becomes greater than 1 if

$$B > 10^5 n_{15} T_4^{-3/2} \Lambda_{10} \text{ G}. \quad (38)$$

Therefore, the current-driven diffusion can dominate the radiative diffusion even at a relatively small current.

The current-driven mechanism leads to a drift of ions in the direction perpendicular to both the magnetic field and electric current. Therefore, a distribution of chemical elements in stars depends essentially on the geometry of fields and currents. In the regions where tangential to the surface components of the both magnetic field and current are greater than normal ones, the considered mechanism may lead to the vertical drift of heavy ions. As a result, surface layers can be overabundant (or underabundant) by heavy element. In the regions where the field is approximately perpendicular to the surface but the current is tangential or the current is normal but the field is tangential, heavy ions drift basically in the tangential direction and can form chemical spots.

The mechanism considered can operate in various astrophysical bodies where the electric currents are non-vanishing. Like other diffusion processes, the current-driven diffusion can lead to a formation of chemical spots if the star has relatively quiescent surface layers. That is the case, for example, for white dwarfs and neutron stars. Many neutron stars have strong magnetic fields and, most likely, topology of these fields is very complex with spot-like structures at the surface (see, e.g., Bonanno et al. 2005, 2006). As it was discussed in this paper, such magnetic structures can be responsible for the formation of a spot-like element distribution at the surface. Such chemical structures can be important, for instance, for the emission spectra, diffusive nuclear burning (Chang & Bildsten 2004, Brown et al. 2002), etc. Evolution of neutron stars is very complicated, particularly, in binary systems (see, e.g., Urpin et al. 1998) and, as a result, a surface chemistry can be complicated as well. Diffusion processes may play an important role in this chemistry (see, e.g., Brown et al. 2002, Medin & Cumming 2014) and can be the reason of chemical sports on the surface of these stars.

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